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AN OBSERVER DESIGN METHOD FOR AN INDUCTION MACHINE

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Summary :
In this study a new method of observer design for a certain class of nonlinear systems is suggested. The method is then applied to an induction machine model to design an observer for flux estimation. In this model, stator current and rotor speed are assumed to be measurable quantities. The observer, by construction, is a discrete time system and it works under small sampling rates as well. In this method of observer design not only can we assign the eigenvalues of the error dynamics but also we can specify all the entries of the error dynamics matrix without any computational burden. Simulated results are presented.

1. Introduction :

As it is well known, the rotor flux should be completely known in order to design good performance controllers for asynchronous machines. Unfortunately, it is very hard to measure the rotor flux and one needs an observer to estimate it. In general, observers for rotor flux estimation have been designed either by using certain approximations on machine models or by reducing the order of the model (see [1],[2],[3],[4],[5]). In this study, first, an exact discrete time equivalent model of a continuous time induction machine model has been obtained. On this equivalent model, a discrete time observer is designed using a special method. This observer differs from the already available induction machine observers in respects that it is 4-dimensional and the rate of decrease of the error can be arbitrarily assigned. If the measurements and system parameters are correctly known, the observer error can be shown to approach zero monotonically.

2. Induction Machine Model :

In this study the following induction machine model is used.

$$\frac{d}{dt}\begin{bmatrix} I_d \\ I_q \\ \lambda_d \\ \lambda_q \end{bmatrix} = A(\omega_s, \omega)\begin{bmatrix} I_d \\ I_q \\ \lambda_d \\ \lambda_q \end{bmatrix} + \bar{b}(V_d, V_q) \qquad (2.1)$$

and

$$\frac{d\omega}{dt} = -\frac{B}{J}\omega + \frac{M^2/J}{L_s L_r - M^2}(x_2 x_3 - x_1 x_4) - \frac{T_L}{J} \qquad (2.2)$$

$$\bar{y}=\begin{bmatrix} I_d \\ I_q \end{bmatrix} \quad , \quad y'=\omega \tag{2.3}$$

where

$$A(\omega_s,\omega)=\begin{bmatrix} -\dfrac{(L_rR_s+M^2\frac{R_r}{L_r})}{L_rL_s-M^2} & \omega_s & \dfrac{M\frac{R_r}{L_r}}{L_rL_s-M^2} & \dfrac{\omega M}{L_rL_s-M^2} \\ -\omega_s & -\dfrac{(L_rR_s+M^2\frac{R_r}{L_r})}{L_rL_s-M^2} & \dfrac{-\omega M}{L_rL_s-M^2} & \dfrac{M\frac{R_r}{L_r}}{L_rL_s-M^2} \\ \dfrac{R_rM}{L_r} & 0 & -\dfrac{R_r}{L_r} & \omega_s-\omega \\ 0 & \dfrac{R_rM}{L_r} & \omega-\omega_s & -\dfrac{R_r}{L_r} \end{bmatrix} \tag{2.4}$$

$$\bar{b}(V_d,V_q)=\begin{bmatrix} \dfrac{L_r}{L_rL_s-M^2}V_d \\ \dfrac{L_R}{L_rL_s-M^2}V_q \\ 0 \\ 0 \end{bmatrix} \tag{2.5}$$

λ_d, λ_q are d and q components of the rotor flux linkeage,
ω_s, ω are stator mmf and rotor angular velocity respectively,
J, B are inertia and viscosity respectively.
V_d, V_q, ω_s and ω are assumed to be inputs in the model. The outputs of the system are taken as I_d and I_q since they can be measured quite easily. The rotor angular velocity ω is also treated as an output variable. In matrix form, then the dynamical system is represented as

$$\dot{\bar{x}}=A(t,\bar{u}(t),\bar{y}(t))\,\bar{x}(t)+\bar{b}(t,\bar{u}(t),\bar{y}(t)) \tag{2.6}$$

$$\bar{y}=\bar{h}'(t,\bar{u}(t),\bar{x}(t))=C(t,\bar{u}(t),\bar{y}(t))\,\bar{x}+\bar{d}(t,\bar{u}(t),\bar{y}(t)) \tag{2.7}$$

Here,

$$\bar{x}=[x_1\ x_2\ x_3\ x_4]^T=[I_d\ I_q\ \lambda_d\ \lambda_q]^T \tag{2.8}$$

$$A(t,\bar{u},\bar{y})=A(\omega_s,\omega)\ ,\quad \bar{b}(t,\bar{u},\bar{y})=\bar{b}(V_d,V_q)\ , \tag{2.9}$$

$$C(t,\bar{u},\bar{y}) = C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} , \quad \bar{d}(t,\bar{u},\bar{y}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2.10}$$

A new discrete time observer design technique for the system represented by the equations (2.6) and (2.7) will be introduced in section 4. The required discrete time equivalent of the system will be obtained in the next section by a special technique which may be called as "the exact discretezation method (EDM)".

3. Exact Discretization Method (EDM) :
Since $A(.,.,.)$, $b(.,.,.)$, $C(.,.,.)$ and $d(.,.,.)$ matrices are known for $t \geq t_0$, the system is considered as linear. Hence the exact solution $\bar{x}(t)$ of the system can be written as

$$\bar{x}(t) = G(t,\bar{u}(t),\bar{y}(t))\bar{x}(t_k) + \bar{h}(t,\bar{u}(t),\bar{y}(t)) \ , \quad t \geq t_k \tag{3.1}$$

from which $\bar{x}(t_{k+1})$ is obtained.

$$\bar{x}(t_{k+1}) = G(t_{k+1},\bar{u}(t_{k+1}),\bar{y}(t_{k+1}))\bar{x}(t_k) + \bar{h}(t_{k+1},\bar{u}(t_{k+1}),\bar{y}(t_{k+1})) \tag{3.2}$$

If the following notation is used

$$\left.\begin{array}{c} \bar{x}_k = \bar{x}(t_k) \ , \quad \bar{y}_k = \bar{y}(t_k) \ , \quad \bar{u}_k = \bar{u}(t_k) \ , \\ C_k = C(t_k,\bar{u}_k,\bar{y}_k) \ , \quad \bar{d}_k = \bar{d}(t_k,\bar{u}_k,\bar{y}_k) \ , \\ G_k = G(t_k,\bar{u}_k,\bar{y}_k) \ , \quad \bar{h}_k = \bar{h}(t_k,\bar{u}_k,\bar{y}_k) \\ k = 0,1,2,\ldots \end{array}\right\} \tag{3.3}$$

then the exact discretized form of the system of equations (2.6) and (2.7) becomes

$$\left.\begin{array}{ll} \bar{x}_{k+1} = G_{k+1}\bar{x}_k + \bar{h}_{k+1} \ , & \bar{x}_0 = \bar{x}(t_0) \\ \bar{y}_k = C_k\bar{x}_k + \bar{d}_k & \end{array}\right\} \tag{3.4}$$

In order to compute G_{k+1} and h_{k+1}, following $n+1$ systems are constructed.

$$\left.\begin{array}{l} \dot{\bar{v}}^i = A(t,\bar{u}(t),\bar{y}(t))\bar{v}^i + \bar{b}(t,\bar{u}(t),\bar{y}(t)) \\ \bar{v}^i(t_k^+) = \bar{v}_{k^\cdot}^i \ , \quad i = 1,2,\ldots,n+1 \end{array}\right\} \tag{3.5}$$

where $\bar{v}^i$ is the state vector of the i-th system and $\bar{v}^i(t) \in \Re^n$ for $i = 1,2,\ldots, n+1$. Defining $\Phi(t)$, $\Phi_{k^\cdot}$ and both in $\Re^{n \times (n+1)}$ by

$$\Phi(t) = [\bar{v}^1(t) \vdots \ldots \vdots \bar{v}^{n+1}(t)] \ , \quad \Phi_{k^\cdot} = [\bar{v}_{k^\cdot}^1 \vdots \ldots \vdots \bar{v}_{k^\cdot}^{n+1}] \tag{3.6}$$

(3.5) is transferred into a matrix differential equation

$$\left.\begin{aligned}\dot{\Phi}(t) &= A(t,\bar{u}(t),\bar{y}(t))\,\Phi(t)\\ &\quad+[\bar{b}(t,\bar{u}(t),\bar{y}(t)) \vdots \ldots \vdots \bar{b}(t,\bar{u}(t),\bar{y}(t))]\,,\\ \Phi(t_k^{+}) &= \Phi_{k^{*}}\,, \qquad t_k \le t \le t_{k+1}\end{aligned}\right\} \tag{3.7}$$

Note that the system of equations (3.7) is initialized at $t = t_k^{+}$ by an arbitrary initial state matrix $\Phi_{k^{*}}$ so that

$$\Phi_k = \Phi(t_k) \neq \Phi(t_k^{+}) = \Phi_{k^{*}}$$

The equations (3.7) are discretized by the EDM to yield

$$\Phi_{k+1} = G_{k+1}\Phi_{k^{*}} + [\bar{h}_{k+1} \vdots \ldots \vdots \bar{h}_{k+1}]\,, \quad \Phi_{k^{*}} = \Phi(t_k^{+}) \tag{3.8}$$

$$\Phi_{k+1} = [G_{k+1} \vdots \bar{h}_{k+1}]\begin{bmatrix} & \Phi_{k^{*}} & \\ \ldots & \ldots & \ldots \\ 1 & \ldots & 1\end{bmatrix} = [G_{k+1} \vdots \bar{h}_{k+1}]\,\bar{\Phi}_{k^{*}} \tag{3.9}$$

The arbitrary extended initial condition matrix $\bar{\Phi}_{k^{*}}$ is chosen as a nonsingular matrix. Then

$$[G_{k+1} \vdots \bar{h}_{k+1}] = \Phi_{k+1}\bar{\Phi}_{k^{*}}^{-1} \tag{3.10}$$

Note that, in practice, the matrix $\bar{\Phi}_{k^{*}}$ will be inverted only once and then used all the computations there after. Furthermore, the system (3.7) need not be initialized at every sampling instant $t = t_k$ using Φ_k instead of $\Phi_{k^{*}}$. But one cannot guarantee that Φ_k will be nonsingular.

4. Discrete Time Observer Design by the EDM :

For a discrete time system,

$$\left.\begin{aligned}\bar{x}_{k+1} &= \bar{f}_d(t_k,\bar{u}_k,\bar{x}_k)\,, \quad \bar{x}_0 \ \text{given}\\ \bar{y}_k &= \bar{h}_d(t_k,\bar{u}_k,\bar{x}_k)\end{aligned}\right\} \tag{4.1}$$

the observer equation is introduced by

$$\bar{z}_{k+1} = M_{k+1}\bar{z}_{k-\alpha+1} + \sum_{i=0}^{\alpha-1} N_{k+1}^{i}\bar{y}_{k-i} + \bar{q}_{k+1} \tag{4.2}$$

where $\bar{z}_k$ is the observer's state at time t_k ($\bar{z}_k \in \Re^n$), M_{k+1} is the given error dynamics matrix such that

$$\overline{e}_k=\overline{z}_k-\overline{x}_k \quad , \quad \overline{e}_{k+1}=M_{k+1}\overline{e}_{k-\alpha+1} \quad ; \tag{4.3}$$

α is the minimum positive integer so that the rank of the matrix

$$Q_\alpha^* = \begin{bmatrix} C_{k-\alpha+1} \\ C_{k-\alpha+2}G_{k-\alpha+2} \\ \vdots \\ C_k\left(\prod_{j=k-\alpha+2}^{k} G_j\right) \end{bmatrix} \tag{4.4}$$

equals n, N^i_{k+1}, i = 0,1,2,...,α-1 are determined from

$$\left[N^{\alpha-1}_{k+1} \vdots N^{\alpha-2}_{k+1} \vdots \ldots \vdots N^0_{k+1}\right] Q^*_\alpha = \left(\prod_{j=k-\alpha+2}^{k+1} G_j\right) - M_{k+1} \tag{4.5}$$

and lastly $\overline{q}_{k+1}$ is given by

$$\left.\begin{aligned} \overline{q}_{k+1} = &- \sum_{i=0}^{\alpha-2} N^i_{k+1}\left(C_{k-i}\left[\sum_{l=-\alpha+3}^{-i+1}\left(\prod_{j=K+l}^{k-i} G_j\right)\overline{h}_{k+l-1}\right]+\overline{d}_{k-i}\right) \\ &+ \left\{\sum_{l=-\alpha+3}^{2}\left(\prod_{j=k+l}^{k+1} G_j\right)\overline{h}_{k+l-1}\right\} \end{aligned}\right\} \tag{4.6}$$

Then it is simple to show that $\lim_{k\to\infty} \|\overline{e}_k\| = 0$ if M_{k+1} is chosen suitably (e.g., a constant diagonal matrix with eigenvalues in the unit circle).

5. Observer for the Induction Machine :

Now, an observer for the induction machine model introduced in section 2 will be constructed using the method explained in section 3. If we denote $I_d, I_q, \lambda_d, \lambda_q$, respectively by x_1, x_2, x_3, x_4 and choose α = 2 with n = 4 , then the observer matrices in section 4 become

$$\left.\begin{aligned} & C_k = C \;, \quad \overline{d}_k = \overline{d} \;, \\ & M_k = M = diag(e^{\lambda T}, e^{\lambda T}, e^{\lambda T}, e^{\lambda T}) \quad , \quad \lambda < 0 \;, \quad T = t_{k+1} - t_k \\ & \left[N^1_{k+1} \vdots\ N^0_{k+1}\right] = (G_{k+1}G_k - M)\begin{bmatrix} C \\ CG_k \end{bmatrix} \\ & q_{k+1} = G_{k+1}\overline{h}_k + \overline{h}_{k+1} - N^0_{k+1}C\overline{h}_k \end{aligned}\right\} \tag{5.1}$$

where G_k and h_k 's are computed for the system given by the equations (2.1) to (2.5) using the method in section 3

6. Numerical Simulations :

The numerical results that will be shown refer to a motor characterized by the following rated values:
V_d = [illegible], V_q = 520 V, $\omega_s = 2\pi 50$ rad/sec, T_L = 5 Nm (for t ≥ 6sec), P = 2, J=1 kgm^2, B = 0.01 Nmsec, $R_s = 0.9\ \Omega$, $R_r = 0.2\ \Omega$, L_s = 0.202 H, L_r = 0.2016 H, M=0.19 H .

The observer is first tested by taking $\lambda = -2$ and T = 0.25 sec with results shown in figure 1.

Because T = 0.25 sec in the first implementation, the 50 Hz oscillations cannot be seen. Of course, one can see that oscillations by taking the sampling rate sufficiently small.

7. Conclusions :

It is possible to operate this observer on real time by using a sufficiently high speed microprocessor-based system which performs the calculations mentioned in the previous sections. The observer output is delayed from the actual system only by the amount of time spent for the computations. On the other hand, this delay is unimportant in observer-based controllers if the controller inputs are changing slowly.

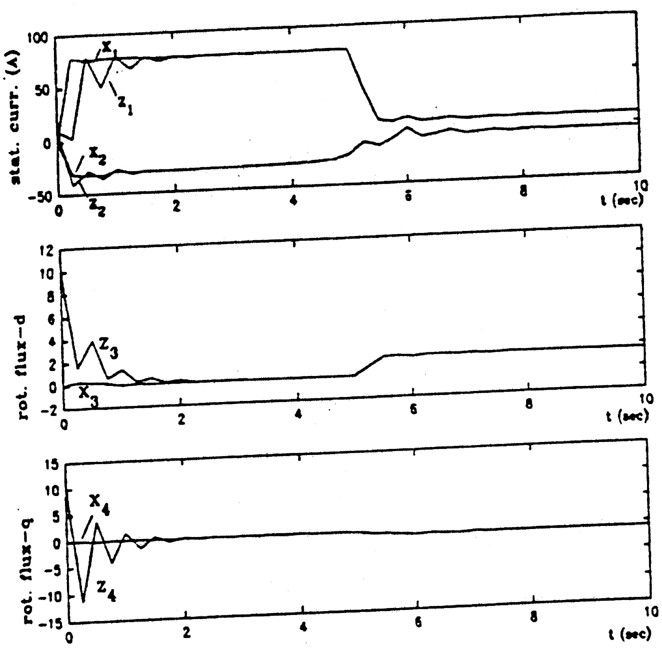

Figure 1

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