

$$A = \begin{bmatrix} 1/2 & -3/2 & 0 \\ -7/6 & -11/6 & -1 \\ 7/4 & 63/12 & 2 \end{bmatrix}$$

$$\lambda_1 = -\frac{1}{3}$$

$$\lambda_2 = \lambda_3 = -\frac{1}{2}$$

①

$$\lambda I - A = \begin{bmatrix} \lambda - \frac{1}{2} & 3/2 & 0 \\ 7/6 & \lambda + 11/6 & 1 \\ -7/4 & -63/12 & \lambda - 2 \end{bmatrix}$$

$$A_{11} = (\lambda - 2)(\lambda + \frac{11}{6}) + \frac{63}{12} = \lambda^2 - \frac{1}{6}\lambda + \frac{19}{12}$$

$$A_{12} = \frac{7}{6}(\lambda - 2) + \frac{7}{4} = \frac{7}{6}\lambda - \frac{7}{12}$$

$$A_{13} = \frac{7}{6} \cdot \frac{-63}{12} + \frac{7}{4}\lambda + \frac{77}{24} = \frac{7}{4}\lambda - \frac{441}{72} + \frac{231}{72} = \frac{7}{4}\lambda - \frac{35}{12}$$

$$A_{21} = \frac{3}{2}\lambda - 3$$

$$A_{22} = \lambda^2 - \frac{5}{2}\lambda + 1$$

$$A_{23} = -\frac{63}{12}\lambda + \frac{63}{24} + \frac{21}{8} = -\frac{63}{12}\lambda + \frac{21}{4}$$

$$A_{31} = \frac{3}{2}$$

$$A_{32} = \lambda - \frac{1}{2}$$

$$A_{33} = \lambda^2 + \frac{4}{3}\lambda - \frac{11}{12} - \frac{21}{12}$$

$$\text{Adj}(A) = \begin{bmatrix} \lambda^2 - \frac{1}{6}\lambda + \frac{19}{12} & -\frac{3}{2}\lambda + 3 & 3/2 \\ -\frac{7}{6}\lambda + \frac{7}{12} & \lambda^2 - \frac{5}{2}\lambda + 1 & -\lambda + \frac{1}{2} \\ \frac{7}{4}\lambda - \frac{35}{12} & \frac{21}{4}\lambda - \frac{21}{4} & \lambda^2 + \frac{4}{3}\lambda - \frac{8}{3} \end{bmatrix}$$

$$\lambda = \frac{1}{3} \Rightarrow \text{Adj}(\lambda I - A) = \begin{bmatrix} 7/4 & 7/2 & 3/2 \\ 35/36 & 35/18 & 5/6 \\ -7/2 & -7 & -3 \end{bmatrix} \quad (2)$$

$$v_1 = \begin{bmatrix} 9 \\ 5 \\ -18 \end{bmatrix}$$

$$\lambda = \frac{1}{2} \Rightarrow \text{Adj}(\lambda I - A) = \begin{bmatrix} 7/4 & 9/4 & 3/2 \\ 0 & 0 & 0 \\ -49/24 & -21/8 & -7/4 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 6 \\ 0 \\ -7 \end{bmatrix} \xrightarrow{\substack{+7 \\ 20}} \begin{bmatrix} +21/10 \\ 0 \\ -49/20 \end{bmatrix} \quad (\lambda I - A) \begin{bmatrix} v_{31} \\ v_{32} \\ v_{33} \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 7 \end{bmatrix}$$

$$\frac{3}{2} v_{32} = -6 \rightarrow v_{32} = -4$$

$$\frac{7}{6} v_{31} + \frac{7}{3} v_{32} + v_{33} = 0 \rightarrow 7v_{31} + 6v_{33} = 56$$

$$-\frac{7}{4} v_{31} - \frac{21}{4} v_{32} - \frac{3}{2} v_{33} = 7 \rightarrow -7v_{31} - 6v_{33} = -56$$

$$-7v_{31} - \underbrace{21v_{32}}_{84} - 6v_{33} = 28$$

$$v_{33} = 0$$

$$v_{31} = 8$$

$$v_3 = \begin{bmatrix} 8 \\ -4 \\ 0 \end{bmatrix}$$

$$V = \begin{bmatrix} 9 & -6 & 8 \\ 5 & 0 & -4 \\ -18 & 7 & 0 \end{bmatrix}$$

$$V^{-1} = \begin{bmatrix} 7/25 & 14/25 & 6/25 \\ -18/25 & -36/25 & -19/25 \\ 7/20 & 9/20 & 3/10 \end{bmatrix}$$

③

$$V^{-1}AV = \begin{bmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 1 \\ 0 & 0 & 1/2 \end{bmatrix} = J$$

$$J^k = \begin{bmatrix} (-1/3)^k & 0 & 0 \\ 0 & (1/2)^k & k(1/2)^{k-1} \\ 0 & 0 & (1/2)^k \end{bmatrix}$$

$$A^k = VJ^kV^{-1}$$

$$A^k = \begin{bmatrix} \frac{63}{25}(-1/3)^k - \frac{38}{25}(1/2)^k + \frac{21}{10}k(1/2)^{k-1} \\ \frac{7}{5}(-1/3)^k - \frac{7}{5}(1/2)^k \\ -\frac{126}{25}(-1/3)^k + \frac{126}{25}(1/2)^k - \frac{49}{20}k(1/2)^{k-1} \end{bmatrix}$$

$$\frac{126}{25}(-1/3)^k - \frac{126}{25}(1/2)^k + \frac{27}{10}k(1/2)^{k-1}$$

$$\frac{14}{5}(-1/3)^k - \frac{9}{5}(1/2)^k$$

$$-\frac{252}{25}(-1/3)^k + \frac{252}{25}(1/2)^k - \frac{63}{20}k(1/2)^{k-1}$$

$$\frac{54}{25}(-1/3)^k - \frac{54}{25}(1/2)^k + \frac{9}{5}k(1/2)^{k-1}$$

$$\frac{6}{5}(-1/3)^k - \frac{6}{5}(1/2)^k$$

$$-\frac{108}{25}(-1/3)^k + \frac{133}{25}(1/2)^k - \frac{21}{10}k(1/2)^{k-1}$$

